ON THE POSSIBILITY OF FASTER-THAN-LIGHT MOTION OF THE COMPTON ELECTRON

G.A. Kotel'nikov

RRC Kurchatov Institute, Kurchatov Sq. 1, Moscow 123182, Russia

E-mail: kga@electronics.kiae.ru

Abstract

The kinematics of Compton-effect with violated invariance of the velocity of light has been considered. It has been shown that in this case faster-than-light motion of the Compton electron is possible. The motion (if it exists in reality) begins with the energy of the incident γ-quantum above 360 keV.

1   Introduction

The Compton scattering is a fundamental effect of nuclear physics [1, 2]. The successive description of its kinematics is essential to any version of the theory. We shall consider the kinematics of this effect in connection with the violation of invariance of the speed of light in the works where the space-time interval takes the form [3] - [5]:

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2, \tag{1}$$

where t is the time; x, y, z are the space variables, $|c| < \infty$ is the velocity of light considered as a variable. It is seen from here that in the space with the metric (1) the event point coordinates are the five numbers: the time t, the space variables x, y, z, and the velocity of light c. Let us denote this space by $V^5(t, \mathbf{x}, c)$. In view of the absence of the space-time variables in an explicit form in front of the differentials dt, dx, dy, dz, the 3-space $R^3(\mathbf{x}) \subset V^5(t, \mathbf{x}, c)$ is homogeneous and isotropic, the time t is homogeneous. This is in agreement with the basic properties of space and time in classical mechanics [6] and Special Relativity (SR) [7] - [9]. Let us suppose that on a particle trajectory the time has a similar property to the universal Newton time in classical physics:

$$dt = dt_0 \rightarrow t = t_0. \tag{2}$$

As a result the velocity of light on the particle trajectory will be depend on the particle velocity by the law

$$c = \pm c_0 \sqrt{1 + v^2/c_0{}^2}, \tag{3}$$

where $c_0 = c'_0 = 3 \cdot 10^{10}$ cm/s is the proper value of the velocity of light. The particle motion perturbs the metric (1), as a result of which the spectrum of c-values is given by the inequality $(c_0 \le |c| < \infty) \subset (|c| < \infty)$. When $v \ne 0$, the metric (1) admits a faster-than-light motion (at the velocity $v > 3 \cdot 10^{10}$ cm/s) of the particle with real mass [3] - [5]. This feature distinguishes the above mentioned publications, and the present work from the well-known theories such as SR [7] - [9], the theory of superluminal motions with imagine mass [14] - [19], the theory of motion with anisotropic tensor of mass [14, 19, 20], and the versions of electrodynamics with instantaneous and retarded interactions [21] - [23]. It is the purpose of the present work to study the kinematics of Compton-effect in space-time with the metric (1) taking into account formula (2) and the positive velocity of light (3).

2 Space-time transformations, group properties

The expression for the interval (1), which we write in the form $ds = F(x, c, dx) > 0$, $dx = (dt, dx, dy, dz)$, possesses the signs inherent in Finsler space: $F(x, c, -dx) = F(x, c, dx) > 0, F(x, c, kdx) = kF(x, c, dx)$, $s = \int F(x, \dot{x}, c)dt = \int(c^2 - \dot{x}^2)^{1/2}dt$, $F(x, k\dot{x}, kc) = kF(x, \dot{x}, c)$, i.e. F is the positively homogeneous function of degree 1 with respect to dt, dx, dy, dz, v and c [10, 11]. By replacing the variables

$$x^0 = \int_0^t cd\tau, \ x^{1,2,3} = x, y, z, \ x^5 = c \tag{4}$$

let us map the space $V^5(t, \mathbf{x}, c)$ with the metric (1) on to the space $F^5(x^0, \mathbf{x}, x^5)$ with the metric

$$ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2, \tag{5}$$

where x^0 will be also considered as re-determined "time" in the case of the particle velocity $v \ne const$. The components of the metric tensor $g_{ab} = (+, -, -, -, 0)$ $(a, b = 0, 1, 2, 3, 5)$ of the space F^5 indicate that F^5, as its subspaces with the metric tensor $g_{\mu\nu} = diag(+, -, -, -)$ $(\mu, \nu = 0, 1, 2, 3)$, includes the Minkowski $M^4{}_1$-space on the hyper-plane $c = c_0$ with the local time $x^0 = c_0 t$; the Minkowski $M^4{}_2$-space with the non-local time (4); zero subspace $R^1{}_0(x^5)$, which coincides with the x^5-axis [12]. (In the $M^4{}_1$-space a point on the x^0-axis corresponds to a point on the t-axis. In the $M^4{}_2$-space a

point on the x^0-axis corresponds to an integral $\int_0^t c(\tau)d\tau$). The infinitesimal space-time transformations, retaining the expression (5) under the condition (2), take the form

$$dx'^{\mu} = L^{\mu}{}_{\nu}dx^{\nu}, \ x'^5 = x^5(1 - \beta \cdot \mathbf{u})/\sqrt{1 - \beta^2}, \ \mu, \nu = 0, 1, 2, 3. \quad (6)$$

Here $L^{\mu}{}_{\nu}$ is the matrix of the Lorentz group L_6, $\beta = \mathbf{V}/c = const$, $\mathbf{u} = \mathbf{v}/c$. For the Lorentz group L_1 and free motions in F^5 and V^5 the corresponding homogeneous integral transformations are

$$x'^0 = \frac{x^0 - \beta x^1}{\sqrt{1 - \beta^2}}, \ x'^1 = \frac{x^1 - \beta x^0}{\sqrt{1 - \beta^2}}, \ x'^{2,3} = x^{2,3}, \ x'^5 = x^5\frac{1 - \beta u^1}{\sqrt{1 - \beta^2}}, \quad (7)$$

$$x' = \frac{x - Vt}{\sqrt{1 - V^2/c^2}}, \ t' = t, \ y' = y, \ z' = z, \ c' = c\frac{1 - Vv_x/c^2}{\sqrt{1 - V^2/c^2}}, \quad (8)$$

where $u^1 = v_x/c$, $v_x = x/t$. They transform into itself the equation of the surface $(x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 = 0 \rightarrow c^2t^2 - x^2 - y^2 - z^2 = 0$ (the zero cone [12] in F^5, the surface of 4-order in V^5). In $M^4{}_1$-space the zero cone changes to the light cone $c_0{}^2t^2 - x^2 - y^2 - z^2 = 0$. The transformations (7) change to the Lorentz transformations. The motions are described by SR [7] - [9]. Let us denote the generator inducing the transformations (7) by $N_{01} = x_0\partial_1 - x_1\partial_0 + u^1x^5\partial_5$ ($N_{01} = ct\partial_x + x\partial_t/c + (x/ct)(c\partial_c - t\partial_t) = ct\partial_x + (x/t)\partial_c$ in the space V^5). It belongs to Lee algebra of the operators $N_{\mu\nu} = x_{\mu}\partial_{\nu} - x_{\nu}\partial_{\mu}$, $Q_0 = (1/t)\partial_c$, $P_0 = (1/c)\partial_t$, $Q_i = \partial_i$, $Z = (c\partial_c - t\partial_t)$, $i = 1, 2, 3$:

$$\begin{aligned}
&[Q_{\mu}, Q_{\nu}] = 0; \\
&[N_{\mu\nu}, N_{\rho\sigma}] = -g_{\mu\rho}N_{\nu\sigma} + g_{\mu\sigma}N_{\nu\rho} + g_{\nu\rho}N_{\mu\sigma} - g_{\nu\sigma}N_{\mu\rho}; \\
&[Q_{\mu}, N_{\nu\rho}] = g_{\mu\nu}Q_{\rho} - g_{\mu\rho}Q_{\nu}; \\
&[P_0, Q_{\nu}] = -g_{0\nu}Z/x_0{}^2; \\
&[P_0, N_{\nu\rho}] = g_{0\nu}P_{\rho} - g_{0\rho}P_{\nu} - (g_{0\rho}x_0 - g_{00}x_{\rho})g_{0\nu}Z/x_0{}^2; \\
&[Z, Q_{\mu}] = [Z, P_0] = [Z, N_{\mu\nu}] = 0;
\end{aligned} \quad (9)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

The algebra, in general case, is infinitely dimensional. As finite subalgebras it includes the algebra of operators $N_{\mu\nu}$ (isomorphic to Lee algebra of Lorentz group [24])), the algebra of operators $N_{\mu\nu}, Q_{\mu}$ (isomorphic to Lee algebra of Poincaré group [24]), the algebra of commutative operators $[Q_{\mu}, Q_{\nu}], [Z, Q_{\mu}], [Z, P_0], [Z, N_{\mu\nu}]$. As a result Lorentz and Poincaré groups arise in the theory not only in the case the speed of light is invariant on the hyper-plane $c = c_0$, but also in the case the time is invariant within the

3

transformations (8) in the V^5-space. Poincaré was first to draw attention to Lorentz group as a symmetry group of the light cone equation $c_0{}^2t^2 - \mathbf{x}^2 = 0$ on the hyper-plane $c = c_0$ [25]. The space of V^5-type and the zero cone $c^2t^2 - \mathbf{x}^2 = 0$ were introduced in the papers [26, 27] in analyzing symmetries of the wave equation with a non-invariant velocity of light.

Let us restrict the consideration of algebra (9) on a set of functions $\phi = \phi(x^0, \mathbf{x}) \subset f(x^0, \mathbf{x}, x^5)$ and take into account $Z\phi = 0$ in this case. The algebra (9) reduces to the Lee algebra of 12-dimensional group $(P_{10}, T_1)X\Delta_1$, where $L_6 \subset P_{10}$ involves hyperbolic rotations on the planes $(x^0, x^i) \subset M^4{}_2$ (the generators $N_{0i} \subset N_{\mu\nu}$), T_4 involves translations along the x^0, x^i axes with t=const (the generators Q_μ), T_1 includes translations along the x^0 axis with c=const (the generator P_0), Δ_1 is the scale transformation of the x^5 axis (generator $Z = x^5\partial_5$). By using the Campbell-Hausdorf formula [28], it can be shown that consecutive operations of Q_0 and P_0 are equivalent to the translation along the x^0 axis: $t' = e^{\theta Q_0}te^{-\theta Q_0} = t + \theta[Q_0, t] + \ldots = t$, $c' = e^{\theta Q_0}ce^{-\theta Q_0} = c + \theta[Q_0, c] + \ldots = c + \theta/t$, $c't' = ct + \theta$; $t'' = e^{\phi P_0}t'e^{-\phi P_0} = t' + \phi[P_0, t'] + \ldots = t' + \phi/c'$, $c'' = e^{\phi P_0}c'e^{-\phi P_0} = c' + \phi[P_0, c'] + \ldots = c'$, $c''t'' = c't' + \phi = ct + \xi$, where $\xi = \theta + \phi$, θ and ϕ are the group parameters. The presence of the P_0 operator corresponds to motion with time if the invariance of the speed of light is violated. Thus, that is impossible within Minkowski $M^4{}_1$-space on the hyper-plane $c = c_0$ is possible within the Minkowski $M^4{}_2$-space entering into the Finsler space with metric (1).

3 Momentum, energy, equations of motion

Let us start from the connections between the partial derivatives:

$$\frac{\partial}{\partial t} = \frac{\partial x^0}{\partial t}\frac{\partial}{\partial x^0} + \frac{\partial x^i}{\partial t}\frac{\partial}{\partial x^i} + \frac{\partial x^5}{\partial t}\frac{\partial}{\partial x^5} = c\frac{\partial}{\partial x^0} \Longrightarrow \frac{\partial}{\partial x^0} = \frac{1}{c}\frac{\partial}{\partial t};$$

$$\frac{\partial}{\partial x} = \frac{\partial x^0}{\partial x}\frac{\partial}{\partial x^0} + \frac{\partial x^i}{\partial x}\frac{\partial}{\partial x^i} + \frac{\partial x^5}{\partial x}\frac{\partial}{\partial x^5} = \frac{\partial}{\partial x^1} \Longrightarrow \frac{\partial}{\partial x^1} = \frac{\partial}{\partial x}; \tag{10}$$

$$\frac{\partial}{\partial c} = \frac{\partial x^0}{\partial c}\frac{\partial}{\partial x^0} + \frac{\partial x^i}{\partial c}\frac{\partial}{\partial x^i} + \frac{\partial x^5}{\partial c}\frac{\partial}{\partial x^5} = \frac{t}{c}\frac{\partial}{\partial t} + \frac{\partial}{\partial x^5} \Longrightarrow x^5\frac{\partial}{\partial x^5} = c\frac{\partial}{\partial c} - t\frac{\partial}{\partial t}.$$

Here the expressions for $\partial/\partial y$ and $\partial/\partial z$ are analogous to $\partial/\partial x$. It is assumed, that the velocity of light does not depend on space variables in the range of interactions - $\nabla c = 0$. As a result the values of the type $(\int_0^t d\tau \partial c/\partial x)\partial/\partial x^0$ vanish. The summing is made over twice repeating index. Then [4]:
- As in SR, the parameter $\beta = V/c$ is in the range of $0 \leq \beta < 1$.
- As in SR, dx^0 is the total differential.

- Generally speaking, the "time" $x^0 = \int_0^t c \, d\tau$ is a functional of $c(\tau)$.
- The condition $\nabla c(x^0) = 0 \leftrightarrow \nabla c(t) = 0$ is invariant on the trajectory of a particle.

Keeping this in the mind, let us construct the theory in the $M^4{}_2$-space which is similar to SR in the $M^4{}_1$-space. By using the relations (10), let us map it on to the V^5-space with the metric (1). Following [8], we start with the integral of action:

$$S = S_m + S_{mf} + S_f = -mc_0 \int ds - \frac{e}{c_0} \int A_\mu dx^\mu - \frac{1}{16\pi c_0} \int F_{\mu\nu} F^{\mu\nu} d^4 x =$$

$$\int \left[-mc_0 \sqrt{1 - u^2} + \frac{e}{c_0}(\mathbf{A} \cdot \mathbf{u} - \phi) \right] dx^0 - \frac{1}{8\pi c_0} \int (E^2 - H^2) d^3 x \, dx^0 =$$

$$-mc_0 \int ds - \frac{1}{c_0} \int A_\mu j^\mu d^4 x - \frac{1}{16\pi c_0} \int F_{\mu\nu} F^{\mu\nu} d^4 x. \qquad (11)$$

Here m is the mass of a particle; e is the electrical charge; $S_m = -mc_0 \int ds = -mc_0 \int \sqrt{1 - u^2} dx^0 = -mc_0 \int (c_0/c) dx^0$ is the action for a free particle; $S_f = -(1/16\pi c_0) \int F_{\mu\nu} F^{\mu\nu} d^4 x$ is the action for a free electromagnetic field; $S_{mf} = -(e/c_0) \int A_\mu dx^\mu = -(1/c_0) \int A_\mu j^\mu d^4 x$ is the action corresponding to the interaction of the charge with electromagnetic field; $A^\mu = (\phi, \mathbf{A})$ is the 4-potential; $A_\mu = g_{\mu\nu} A^\nu = (\phi, -\mathbf{A})$; $j^\mu = (\rho, \rho\mathbf{u})$ is the 4-vector of the density of a current; ρ is the density of the charge; $\mathbf{u} = \mathbf{v}/c$ is the dimensionless 3-velocity of a particle; $\mathbf{E} = -\partial\mathbf{A}/\partial x^0 - \nabla\phi$ is the electric field; $\mathbf{H} = \nabla X \mathbf{A}$ is the magnetic field; $F_{\mu\nu} = \partial A_\nu / \partial x^\mu - \partial A_\mu / \partial x^\nu$ is the tensor of an electromagnetic field; $F_{\mu\nu} F^{\mu\nu} = 2(H^2 - E^2)$; $d^4 x = dx^0 dx^1 dx^2 dx^3$ is the element of the invariant 4-volume. The speed of light c_0, the mass of a particle m, the electrical charge e are invariant constants of the theory.

In spite of the fact that the action (11) is similar to the action of SR, it differs from the SR action [8]. The electrical field has been chosen in the form $\mathbf{E} = -\partial\mathbf{A}/\partial x^0 - \nabla\phi = -(1/c)\partial\mathbf{A}/\partial t - \nabla\phi$ instead of $\mathbf{E} = -(1/c_0)\partial\mathbf{A}/\partial t - \nabla\phi$ [8] [8]. The current density has been chosen in the form $j^\mu = (\rho, \rho\mathbf{u}) = (\rho, \rho\mathbf{v}/c)$ instead of $j^\mu = (\rho, \rho\mathbf{v})$ [8]. The current density is similar to the expression from Pauli monograph [7] with the only difference that $\mathbf{j}$ in (11) is equal to $\rho\mathbf{v}/c$ instead of $\rho\mathbf{v}/c_0$ [7]. Analogously, the propagation velocity of the 4-potential in (11) is equal to c instead of c_0 [8]. The action (11) goes into the SR action, if we replace c by c_0 within the corresponding expressions. In accordance with the construction, the action (11) is Lorentz invariant and does not depend on the x^5 variable. As a result the action (11) is invariant with respect to the group $(P_{10}, T_1) X \Delta_1$, induced by the reduction of the algebra (9) on the set of functions $\phi = \phi(x^0, \mathbf{x})$.

Lagrangian L, the generalized momentum $\mathbf{P}$ and the generalized energy $\mathcal{H}$ take the form:

$$L = -mc_0\sqrt{1 - u^2} + \frac{e}{c_0}(\mathbf{A} \cdot \mathbf{u} - \phi); \tag{12}$$

$$\mathbf{P} = \frac{\partial L}{\partial \mathbf{u}} = \frac{mc_0\mathbf{u}}{\sqrt{1 - u^2}} + \frac{e}{c_0}\mathbf{A} = \mathbf{p} + \frac{e}{c_0}\mathbf{A} = m\mathbf{v} + \frac{e}{c_0}\mathbf{A}; \tag{13}$$

$$\mathcal{H} = \mathbf{P} \cdot \mathbf{u} - L = \frac{mc_0c + e\phi}{c_0} = \frac{\mathcal{E} + e\phi}{c_0}. \tag{14}$$

$$\mathbf{p} = m\mathbf{v}, \quad \mathcal{E} = mcc_0 = mc_0{}^2\sqrt{1 + \frac{v^2}{c_0{}^2}}. \tag{15}$$

Here $\mathbf{p}$, $\mathcal{E}$ are the momentum and energy of a particle with mass m. They may be combined into 4- momentum p^μ

$$p^\mu = mc_0u^\mu = \left(\frac{mc_0c}{c_0}, mcu^i\right) = \left(\frac{\mathcal{E}}{c_0}, m\mathbf{v}\right), \tag{16}$$

the components of which are related as follows:

$$p_\mu p^\mu = \frac{\mathcal{E}^2}{c_0{}^2} - \mathbf{p}^2 = m^2c_0{}^2; \quad \mathbf{p} = \frac{\mathcal{E}}{c_0c}\mathbf{v}; \quad \mathbf{p} = \frac{\mathcal{E}}{c_0c}\mathbf{c}, \; if \; m = 0, \; \mathbf{v} = \mathbf{c}. \tag{17}$$

It is seen from here that the momentum of a particle with the mass $m = 0$ is independent of the absolute value of the particle velocity $v = c$. It is determined only by the energy of a particle: $\mathbf{p} = \mathbf{n}\mathcal{E}/c_0$, $\mathbf{n} = \mathbf{c}/c$. (In SR this property is masked by c_0 being constant).

Next, let us start from the mechanical [8] and the field equations [30] of Lagrange

$$\frac{d}{dx^0}\frac{\partial L}{\partial \mathbf{u}} - \frac{\partial L}{\partial \mathbf{x}} = 0; \quad \frac{\partial}{\partial x^\nu}\frac{\partial \mathcal{L}}{\partial(\partial A_\mu/\partial x^\nu)} - \frac{\partial \mathcal{L}}{\partial A_\mu} = 0, \tag{18}$$

where L is the Lagrangian (12), $\mathcal{L} = -(1/c_0)A_\mu j^\mu - (1/16\pi c_0)F_{\mu\nu}F^{\mu\nu}$ is the density of Lagrange function for electromagnetic field and interaction between the field and the charge. Taking into consideration the equality $\nabla(\mathbf{a} \cdot \mathbf{b}) = (\mathbf{a} \cdot \nabla)\mathbf{b} + (\mathbf{b} \cdot \nabla)\mathbf{a} + \mathbf{a}\mathrm{x}(\nabla\mathrm{x}\mathbf{b}) + \mathbf{b}\mathrm{x}(\nabla\mathrm{x}\mathbf{a})$, the permutable relationships for the tensor of electromagnetic field, the expression $\partial(F_{\mu\nu}F^{\mu\nu})/\partial(\partial A_\mu/\partial x^\nu) = -4F^{\mu\nu}$ [8], we find the equations of motions of electromagnetic field and of

6

a particle in the field

$$\frac{d\mathbf{p}}{dx^0} = \frac{e}{c_0}\mathbf{E} + \frac{e}{c_0}\mathbf{u}\mathbf{x}\mathbf{H}; \quad \frac{d\mathcal{E}}{dx^0} = e\mathbf{E}\cdot\mathbf{u};$$

$$\frac{\partial F_{\mu\nu}}{\partial x^\rho} + \frac{\partial F_{\nu\rho}}{\partial x^\mu} + \frac{\partial F_{\rho\mu}}{\partial x^\nu} = 0; \quad \frac{\partial F^{\mu\nu}}{\partial x^\nu} + 4\pi j^\mu = 0. \tag{19}$$

(Here $\mathbf{p} = mc_0\mathbf{u}/\sqrt{1-u^2}$, $\mathcal{E} = mc_0{}^2/\sqrt{1-u^2}$). In the variables (x^0, x^1, x^2, x^3) equations (19) coincide exactly with the equations [8] and are the same for both the Minkowski spaces - $M^4{}_1$ and $M^4{}_2$. The difference arises if the equations are written with the variables (t, x, y, z). In the case of $M^4{}_1$-Minkowski space the equations coincide with SR equations [8], if we put $c = c_0$, $dx^0 = c_0 dt$ into them. (In accordance with going the action (11) into the SR action [8]). In the case of $M^4{}_2$-Minkowski space it is necessary to take into account $dx^0 = cdt$, $\sqrt{1-u^2} = c_0/c$, and the relations (10). Then the equations of motions take the forms [3] - [5]:

$$\frac{d\mathbf{p}}{dt} = m\frac{d\mathbf{v}}{dt} = \frac{c}{c_0}e\mathbf{E} + \frac{e}{c_0}\mathbf{v}\mathbf{x}\mathbf{H}; \quad \frac{d\mathcal{E}}{dt} = e\mathbf{E}\cdot\mathbf{v} \rightarrow m\frac{dc}{dt} = \frac{e}{c_0}\mathbf{v}\cdot\mathbf{E}. \tag{20}$$

$$\nabla\mathbf{X}\mathbf{E} + \frac{1}{c}\frac{\partial\mathbf{H}}{\partial t} = 0; \qquad \nabla\cdot\mathbf{E} = 4\pi\rho;$$

$$\nabla\mathbf{X}\mathbf{H} - \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} = 4\pi\rho\frac{\mathbf{v}}{c}; \quad \nabla\cdot\mathbf{H} = 0, \tag{21}$$

where $c(t) = c_0(1 + v^2/c_0{}^2)^{1/2} = c(0)[1 + (e/mc_0c(0))\int_0^t \mathbf{v}\cdot\mathbf{E}d\tau]$, $\nabla c = 0$. Equations (20) - (21), if considered as the whole, form a set of the self-consistent nonlinear equations. (In the approximation $v^2/c_0{}^2 \ll 1$ by $c \sim c_0$, they describe the motion of non-relativistic particle in electromagnetic field and coincide with [29]). They admit faster-than-light motion of a particle with the real mass m, rest energy $\mathcal{E}_0 = mc_0{}^2$ and the velocity

$$v = \sqrt{\mathcal{E}^2 - m^2c_0{}^4}/mc_0 > c_0, \tag{22}$$

if the energy of a particle satisfies the inequality $\mathcal{E} > \sqrt{2}\mathcal{E}_0$. For example, for the proton the rest energy is equal 938 MeV. The 1 GeV proton velocity is about $0.37c_0$. Faster-than-light motion of the proton begins with the energy ~ 1.33 GeV. The faster-than-light electron motion ($\mathcal{E}_0 = 511$ keV) begins with the energy ~ 723 keV. The calculated velocity of 1 GeV electron is $\sim 2000\ c_0$. Thus, if $M^4{}_2$-Minkowski space were realized in the nature, the neutron physics of nuclear reactors could be formulated in the approximation

7

$v \ll c_0$, as in SR. The particle physics on modern accelerators would be the physics of faster-than-light motions. The results obtained are given in Table 1 in comparison with the analogous results from classical mechanics and SR. In this Table the designations are used: $d\mathbf{x}^2 = dx^2 + dy^2 + dz^2$, T is the kinetic energy, $\beta = V/c$.

Table 1

The classical mechanics [6]	Special Relativity [7] − [9]	Present work [3] − [5]
$ds^2 = d\mathbf{x}^2$	$ds^2 = c_0{}^2 dt^2 - d\mathbf{x}^2$	$ds^2 = c^2 dt^2 - d\mathbf{x}^2$
$x' = x - Vt,$ $y' = y,\ z' = z,$ $t' = t,$ $c' = c\sqrt{1 - 2\beta n_x + \beta^2}$	$x' = \dfrac{x - Vt}{\sqrt{1 - \beta^2}},$ $y' = y,\ z' = z,$ $t' = \dfrac{t - Vx/c_0{}^2}{\sqrt{1 - \beta^2}},$ $c_0' = c_0$	$x' = \dfrac{x - Vt}{\sqrt{1 - \beta^2}},$ $y' = y,\ z' = z,$ $t' = t,$ $c' = c\dfrac{1 - Vv_x/c^2}{\sqrt{1 - \beta^2}}$
$\mathbf{p} = m\mathbf{v}$ $T = \dfrac{mv^2}{2}$ $T = \dfrac{p^2}{2m}$ $m\dfrac{d\mathbf{v}}{dt} = e\mathbf{E} + \dfrac{e}{c_0}\mathbf{v}\text{x}\mathbf{H}[29]$ $\dfrac{dT}{dt} = e\mathbf{v}\cdot\mathbf{E}$	$\mathbf{p} = \dfrac{m\mathbf{v}}{\sqrt{1 - v^2/c_0{}^2}}$ $\mathcal{E} = \dfrac{mc_0{}^2}{\sqrt{1 - v^2/c_0{}^2}}$ $\mathcal{E}^2 - c_0{}^2 p^2 = m^2 c_0{}^4$ $\dfrac{d\mathbf{p}}{dt} = e\mathbf{E} + \dfrac{e}{c_0}\mathbf{v}\text{x}\mathbf{H}$ $\dfrac{d\mathcal{E}}{dt} = e\mathbf{v}\cdot\mathbf{E}$	$\mathbf{p} = m\mathbf{v}$ $\mathcal{E} = mc_0{}^2\sqrt{1 + v^2/c_0{}^2}$ $\mathcal{E}^2 - c_0{}^2 p^2 = m^2 c_0{}^4$ $m\dfrac{d\mathbf{v}}{dt} = \dfrac{c}{c_0}e\mathbf{E} + \dfrac{e}{c_0}\mathbf{v}\text{x}\mathbf{H}$ $\dfrac{d\mathcal{E}}{dt} = e\mathbf{v}\cdot\mathbf{E}$

It is shown in [3] - [5], how a lot of experiments (interpreted only by SR until the present time) may be explained with the help of the proposed theory. For example, these are the experiments of Michelson and Fizeau, aberration of light, the appearance of atmospheric μ-mesons on the Earth surface, Doppler-effect, a number of the known experiments for the proof of independence of the speed of light from the emitter velocity, decay of unstable particles, generation of new particles in nuclear reactions, Compton-effect, photo-effect. We consider the kinematics of Compton-effect in more detail.

8

4 Motion integrals: momentum and energy

Let us note that dx^0, according to the construction, is the total differential. As a result x^0 possess property of the time for $M^4{}_2$-Minkowski space. Therefore, by virtue of Lagrange mechanical equations, the momentum and energy (15) for an isolated system are the integrals of motion because of the homogeneity of space-time [6, 8]. The formula for the kinetic energy takes the form

$$T = \mathcal{E} - mc_0{}^2 = mc_0{}^2(\frac{c}{c_0} - 1) = mc_0{}^2\left(\sqrt{1 + \frac{v^2}{c_0{}^2}} - 1\right) \approx \frac{1}{2}mv^2. \quad (23)$$

With $v^2 \ll c_0{}^2$ expression (23) coincides with the expression for kinetic energy in classical mechanics (as in SR). Variations of $\mathcal{E}$ and $\mathbf{p}$ with time determine the dynamics of a particle for $M^4{}_2$-Minkowski space. With $v^2 \ll c_0{}^2$ the new dynamics goes into the Newton dynamics.

Let us use the expressions (15) and (23) to describe the motion of a lot of number of particles. Following [2], we shall consider the reaction in which the particles with masses $m'_1, m'_2, \ldots, m'_n$ are produced in colliding the moving particle m_1 with the immobile particle m_2 (the target). Let us write the conservation laws in the form

$$\mathbf{p}_1 = \mathbf{p}'_1 + \mathbf{p}'_2 + \ldots + \mathbf{p}'_n,$$
$$\mathcal{E}_1 + m_2c_0{}^2 = \mathcal{E}'_1 + \mathcal{E}'_2 + \ldots + \mathcal{E}'_n, \quad (24)$$

where the momentum and energy of each of the particle are given by the formulas (15) ($\mathbf{p}'_i = m'_i\mathbf{v}'_i$, $\mathcal{E}'_i = m'_ic'_ic_0$). By using the relationship between the momentum and energy $\mathcal{E}_1{}^2 = c_0{}^2\mathbf{p}_1{}^2 + m_1{}^2c_0{}^4$ and the property of invariance of the expression $(\sum_i \mathcal{E}_i)^2 - c_0{}^2(\sum_i \mathbf{p}_i)^2 = inv$, we may write the expression of the threshold energy $\mathcal{E}_{1,thr}$ of reaction (24) in the form

$$(\mathcal{E}_{1,thr} + m_2c_0{}^2)^2 - c_0{}^2\mathbf{p}_1{}^2 = (\sum_i m'_i)^2c_0{}^4. \quad (25)$$

From here we find the threshold kinetic energy

$$T_{1,thr} = \frac{(\sum_i m'_i + m_1 + m_2)(\sum_i m'_i - m_1 - m_2)}{2m_2}c_0{}^2. \quad (26)$$

It coincides with the similar formula from SR [2]. The difference arises in calculating the velocity of a hitting particle and the threshold velocity of the

reaction products. Taking into account $\mathcal{E}_{1,thr} = T_{1,thr} + m_1 c_0{}^2$, we find the threshold velocity of the particle m_1:

$$v_1 = c_0 \sqrt{\left[1 + \frac{(\sum_i m'_i + m_1 + m_2)(\sum_i m'_i - m_1 - m_2)}{2 m_1 m_2}\right]^2 - 1}. \qquad (27)$$

It follows from the momentum-energy conservation law (24) that the velocity of the conglomerate of particles $\sum_i m'_i$ moving at the same (threshold) velocity V', will be equal

$$V' = \frac{p_1}{\sum_i m'_i} = \frac{m_1}{\sum_i m'_i} v_1 \qquad (28)$$

In the case of proton-proton collision $p^+ + p^+ = p^+ + p^+ + p^+ + p^-$, when $m_1 = m_2 = m'_i = m_p$, we obtain that the threshold energy for creating the antiproton is equal $\mathcal{E}_{1,thr} = 7 m_p c_0{}^2 \sim 6.6$ GeV in accordance with [2], and $v_1 = \sqrt{48} c_0 \sim 6.9 c_0$, $V' = \sqrt{3} c_0 \sim 1.7 c_0$ in accordance with [4]. In SR these values are equal $\mathcal{E}_{1,thr} = 7 m_p c_0{}^2 \sim 6.6$ GeV [2], $v_1 \to w_1 = \sqrt{48/49} c_0 \sim 0.99 c_0$, $V' \to W' = \sqrt{3/4} c_0 \sim 0.87 c_0$ respectively.

5 Consequences of momentum-energy conservation law for Compton-effect

Let us consider the kinematics of γ-quantum scattering on a free electron with the rest energy $\mathcal{E}_0 = m c_0{}^2$, where m is the mass of electron. By using the momentum-energy conservation law and without concretizing the expressions for the momentum $\mathbf{p}'$ and energy $\mathcal{E}'$ of the scattered electron, we find

$$\begin{aligned}
\hbar\omega + \mathcal{E}_0 &= \hbar\omega' + \mathcal{E}'; \\
\frac{\hbar\omega}{c_0} &= \frac{\hbar\omega'}{c_0} cos\theta + p' cos\alpha; \\
0 &= \frac{\hbar\omega'}{c_0} sin\theta - p' sin\alpha.
\end{aligned} \qquad (29)$$

Here $\hbar$ is the Planck constant, ω and ω' are the frequencies of the incident and scattered γ-quanta, $\hbar\omega$ and $\hbar\omega'$ are the energies of these quanta. The momentum of the incident γ-quantum is directed along the α x-axis, θ is the scattering angle of γ'-quantum, α is the scattering angle of electron e'. The angle θ is counted counterclockwise; the angle α is counted clockwise. Let us rewrite the momentum conservation law in the form

$$p'^2 cos^2\alpha = \left(\frac{\hbar\omega}{c_0} - \frac{\hbar\omega'}{c_0} cos\theta\right)^2, \quad p'^2 sin^2\alpha = \left(\frac{\hbar\omega'}{c_0}\right)^2 sin^2\theta, \qquad (30)$$

10

and square this. By summing the result obtained and by using the conservation energy law and the dispersion expression (17), we find the known formula for the scattered γ'-quantum angular distribution and its frequency [1, 2]

$$\omega' = \frac{\omega}{1 + \frac{\hbar\omega}{\mathcal{E}_0}(1 - cos\theta)}. \tag{31}$$

With the help of (31) we find the scattered γ'-quantum momentum:

$$\mathbf{p'}_\gamma = \frac{\hbar\omega'}{c_0}(cos\theta, sin\theta) = \frac{\hbar\omega}{c_0[1 + \frac{\hbar\omega}{\mathcal{E}_0}(1 - cos\theta)]}(cos\theta, sin\theta). \tag{32}$$

The scattered electron momentum may be found by means of putting (31) into (30):

$$p' cos\alpha = \frac{\hbar\omega}{\mathcal{E}_0}mc_0\frac{(\mathcal{E}_0 + \hbar\omega)(1 - cos\theta)}{\mathcal{E}_0 + \hbar\omega(1 - cos\theta)};$$
$$p' sin\alpha = \frac{\hbar\omega}{\mathcal{E}_0}mc_0\frac{\mathcal{E}_0 sin\theta}{\mathcal{E}_0 + \hbar\omega(1 - cos\theta)}. \tag{33}$$

$$p'(\theta) = \frac{\hbar\omega}{\mathcal{E}_0}mc_0\frac{\sqrt{\mathcal{E}_0^2 sin^2\theta + (\mathcal{E}_0 + \hbar\omega)^2(1 - cos\theta)^2}}{\mathcal{E}_0 + \hbar\omega(1 - cos\theta)}. \tag{34}$$

The relationship between the scattered electron angle and the scattered γ'-quantum angle may be derived from (33) and takes the form

$$tg\alpha = \frac{\mathcal{E}_0 sin\theta}{(\mathcal{E}_0 + \hbar\omega)(1 - cos\theta)}. \tag{35}$$

The equality $\alpha = 0$ induces the solutions $\theta = \pm k\pi$, $k = 0, 1, 2, \ldots$, which corresponds to propagation of the scattered γ'-quantum along and opposite the direction of moving the Compton electron. Suppose $\theta = 0$ and $\theta = \pi$, we find the expressions for the forward scattered electron momentum p' with $\alpha = 0$:

$$\alpha = 0; \quad \theta = 0; \quad \omega' = \omega; \qquad p'_{\theta=0} = 0;$$
$$\alpha = 0; \quad \theta = \pi; \quad \omega' = \frac{\omega}{1 + \frac{2\hbar\omega}{\mathcal{E}_0}}; \quad p'_{\theta=\pi} = \frac{\hbar\omega}{\mathcal{E}_0}mc_0\left[1 + \frac{\mathcal{E}_0}{\mathcal{E}_0 + 2\hbar\omega}\right]. \tag{36}$$

The Compton-electron energy may be found by putting the formula (34)

11

into the dispersion relationship (17):

$$\mathcal{E}' = \sqrt{\mathcal{E}_0{}^2 + \hbar^2\omega^2\frac{\mathcal{E}_0{}^2 sin^2\theta + (\mathcal{E}_0 + \hbar\omega)^2(1 - cos\theta)^2}{[\mathcal{E}_0 + \hbar\omega(1 - cos\theta)]^2}} = \\ \mathcal{E}_0 + \frac{\hbar^2\omega^2(1 - cos\theta)}{\mathcal{E}_0 + \hbar\omega(1 - cos\theta)}. \tag{37}$$

The second, simple form of this formula was derived by using the energy conservation law (29) taking into account the frequency ω' from (31). It is essential that all the results obtained are independent of concrete expressions for the energy and momentum ($\mathcal{E} = mc_0{}^2/\sqrt{1 - v^2/c_0{}^2}$, $\mathbf{p} = m\mathbf{v}/\sqrt{1 - v^2/c_0{}^2}$ for $M^4{}_1$; $\mathcal{E} = mc_0{}^2\sqrt{1 + v^2/c_0{}^2}$, $\mathbf{p} = m\mathbf{v}$ for $M^4{}_2$). Therefore, in view of the laws of conservation (29) and the dispersion relationship (17), these are common to both the Minkowski spaces. The distinctions arise when the transformational properties of the time "t" for the $M^4{}_1$ and $M^4{}_2$-spaces are taken into account in calculating the velocities of the scattered γ'-quantum and Compton electron. By using formula (34), we find that in the $M^4{}_2$-space the Compton electron velocity is

$$v'(\theta) = \frac{\hbar\omega}{\mathcal{E}_0}c_0\frac{\sqrt{\mathcal{E}_0{}^2 sin^2\theta + (\mathcal{E}_0 + \hbar\omega)^2(1 - cos\theta)^2}}{\mathcal{E}_0 + \hbar\omega(1 - cos\theta)}. \tag{38}$$

It is equal to zero with $\theta = 0$. When $\theta = \pi$, the electron velocity will exceed the speed of light c_0 if the following inequality holds:

$$v'(\alpha = 0, \theta = \pi) = c_0\frac{\hbar\omega}{\mathcal{E}_0}\frac{2(\mathcal{E}_0 + \hbar\omega)}{\mathcal{E}_0 + 2\hbar\omega} > c_0. \tag{39}$$

According to (39), faster-than-light motion of the forward-scattered electron begins from the energy of the incident γ-quantum:

$$\hbar\omega > \frac{\mathcal{E}_0}{\sqrt{2}} \sim 360 \text{ keV}. \tag{40}$$

Thus, it follows from the kinematics of Compton-effect that in scattering the γ-quantum in the $M^4{}_2$-Minkowski space, the appearance of electron faster-than-light motion is possible. This motion begins from the γ-quantum energy exceeding 360 keV. For going to SR, the following relations may be used:

$$v' = \frac{w'}{\sqrt{1 - w'^2/c_0{}^2}}, \ w' = \frac{v'}{\sqrt{1 + v'^2/c_0{}^2}}, \tag{41}$$

12

where v' is the velocity of Compton electron in the $M^4{}_2$-space, w' is the velocity of Compton electron in the $M^4{}_1$-space:

$$w'(\theta) = \frac{\hbar\omega}{\mathcal{E}_0}c_0\sqrt{\frac{\mathcal{E}_0{}^2}{\hbar^2\omega^2 + \frac{\mathcal{E}_0{}^2[\mathcal{E}_0+\hbar\omega(1-cos\theta)]^2}{[\mathcal{E}_0{}^2 sin^2\theta+(\mathcal{E}_0+\hbar\omega)^2(1-cos\theta)^2]}}} < c_0. \qquad (42)$$

The relations (41) correspond to the equality of the scattered electron energy in both the Minkowski spaces.

To calculate the scattered γ'-quantum velocity in the $M^4{}_2$-space, the use of the energy-momentum conservation law is scarce. Certain assumptions of the nature of scattering are necessary.

# 6 The possible mechanisms of Compton scattering

6.1 Local scattering

Suppose, an incident γ-quantum is scattered by an immobile electron in the point of its localization in accordance with the Feynman diagram corresponding to the process

$$\gamma + e^- \to \gamma' + e^{-'}.$$

(The thin lines correspond to γ-quanta, the bold line correspond to electrons). As a result of the interaction the scattered γ'-quantum velocity will be equal $c' = c_0 = 3 \cdot 10^{10}$ cm/s and independent of the scattering angle θ in accordance with $c' = c_0\sqrt{1 + v^2/c_0{}^2}$, if $v = 0$. The forward scattered γ'-quantum velocity ($\theta = 0$), as well as the back scattered γ'-quantum velocity ($\theta = \pi$) will be equal the same value of $3 \cdot 10^{10}$ sm/s. The electron gains the velocity (38), becoming the electron $e^{-'}$.

6.2 Non-local scattering. Version A

According to quantum electrodynamics concepts [1, 2], suppose the scattering is described by the Feynman diagram corresponding to the process
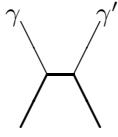
$$\gamma + e^- \rightarrow (e^-)_v \rightarrow \gamma' + e^{-\prime}.$$

The incident γ-quantum is absorbed by the immobile electron in some point of space-time, after which an intermediate state is formed, the virtual electron e^-_v. Next the virtual electron emits the γ'-quantum in another point of space-time and becomes the free scattered electron $e^{-\prime}$. By determining the electron mass m_v and the virtual electron velocity v_v from the energy-momentum conservation law [1]

$$\hbar\omega + \mathcal{E}_0 = m_v c_0^2 (1 + v_v^2/c_0^2)^{1/2}, \ \hbar\omega/c_0 = m_v v_v, \tag{43}$$

we find the expression for the virtual electron mass and the its velocity

$$m_v = \frac{\sqrt{\mathcal{E}_0^2 + 2\hbar\omega\mathcal{E}_0}}{c_0^2};$$

$$v_v = c_0 \frac{\hbar\omega}{\sqrt{\mathcal{E}_0^2 + 2\hbar\omega\mathcal{E}_0}} \sim c_0\sqrt{\frac{\hbar\omega}{2\mathcal{E}_0}}, \text{if } \hbar\omega \gg \mathcal{E}_0. \tag{44}$$

The scattered γ'-quantum velocity will be equal

$$c' = c_0\sqrt{1 + \frac{v_v^2}{c_0^2}} = c_0\sqrt{1 + \frac{\hbar^2\omega^2}{\mathcal{E}_0^2 + 2\hbar\omega\mathcal{E}_0}}. \tag{45}$$

It does not depend on the scattering angle θ of γ'-quantum and with $\hbar\omega \gg \mathcal{E}_0$ is equal $c' \sim c_0\sqrt{\hbar\omega/2\mathcal{E}_0} > c_0$.

6.3 Non-local scattering. Version B

Let us note that the set of equations with a virtual electron admits another mechanism of Compton scattering. Suppose the virtual electron e^-_v transmutes spontaneously into the free electron $e^{-\prime}$ that emits the γ'-quantum

---

[1] The full set of equations with participation of the virtual electron may be written as follows:
$$\hbar\omega + \mathcal{E}_0 = m_v c_0^2 (1 + v_v^2/c_0^2)^{1/2}, \ \hbar\omega/c_0 = m_v v_v;$$
$$m_v c_0^2 (1 + v_v^2/c_0^2)^{1/2} = \hbar\omega' + \mathcal{E}_0(1 + v'^2/c_0^2)^{1/2};$$
$$m_v v_v = (\hbar\omega'/c_0)cos\theta + p'cos\alpha; \ 0 = (\hbar\omega'/c_0)sin\theta - p'sin\alpha.$$

By eliminating $m_v c_0^2 (1 + v_v^2/c_0^2)^{1/2}$ and $m_v v_v$, this set may be reduced to set (29).

14

(the scattered gamma-quantum). In the $M^4{}_1$-space both the mechanisms lead to the same result because the speed of light is constant. In the $M^4{}_2$-space distinctions between A and B-versions are more essential. Indeed, in the case of B the scattered γ'-quantum velocity will be determined by the expression

$$c' = c_0\sqrt{1 + \frac{v'^2}{c_0{}^2}} = c_0\sqrt{1 + \left(\frac{\hbar\omega}{\mathcal{E}_0}\right)^2 \frac{\mathcal{E}_0{}^2 sin^2\theta + (\mathcal{E}_0 + \hbar\omega)^2(1 - cos\theta)^2}{[\mathcal{E}_0 + \hbar\omega(1 - cos\theta)]^2}} \quad (46)$$

instead of the formula (45), because the Compton electron velocity (38) differs from the virtual electron velocity (44). As a result the scattered γ'-quantum velocity comes to depend on the angle of its scattering. In the case of forward-scattering with $\alpha = 0$, $\theta = 0$ this velocity is minimal and equal $c' = c_0$. The γ'-quantum energy is maximal and coincides, according to (31), with the incident γ-quantum energy $\hbar\omega' = \hbar\omega$. With the small scattering angles when $sin\theta \sim \theta$, $cos\theta \sim (1 - \theta^2/2)$ the scattered γ'-quantum velocity is $c' \sim c_0\{1 + (\hbar\omega/\mathcal{E}_0)^2[\mathcal{E}_0{}^2\theta^2 + (\hbar\omega)^2\theta^4/4]/[\mathcal{E}_0 + \hbar\omega\theta^2/2]^2\}^{1/2}$, if $\hbar\omega \gg \mathcal{E}_0$. With $\theta \ll \mathcal{E}_0/\hbar\omega$ it is equal $c' \sim c_0[1 + (\hbar\omega\theta/\mathcal{E}_0)^2/2] \sim c_0$. In the case of back-scattering with $(\theta \sim \pi)$ the scattered γ'-quantum energy is minimal $\hbar\omega' \sim \hbar\omega/(1 + 2\hbar\omega/\mathcal{E}_0) \sim \mathcal{E}_0/2$, but its velocity is maximal $c' \sim c_0(\hbar\omega/\mathcal{E}_0)$, if $\hbar\omega \gg \mathcal{E}_0$.

7 Comparison of the kinematics of Compton - effect within the $M^4{}_1$ and $M^4{}_2$-spaces

In sum, we can note the following features of Compton-effect kinematics within the Minkowski spaces $(M^4{}_1, M^4{}_2) \subset F^5$.

- The expression for the scattered γ'-quantum frequency ω' in the $M^4{}_2$-space coincides with the similar expression for the scattered γ'-quantum frequency in the $M^4{}_1$-space (as in SR).

- The expressions for the scattered electron momentum and energy and for the scattered γ'-quantum momentum and energy in $M^4{}_2$ coincide with the similar expressions within $M^4{}_1$.

- The distinctions arise in calculating the velocities of the scattered γ'-quantum and scattered electron. Within $M^4{}_1$ the velocity of scattered quantum is always equal $c_0 = 3 \cdot 10^{10}$ cm/s. The Compton electron velocity does not exceed c_0.

15

- In $M^4{}_2$ in scattering the incident γ-quantum by the immobile electron in the point of its localization, the scattered γ'-quantum velocity does not depend on the scattering angle and is equal $c_0 = 3 \cdot 10^{10}$ cm/s (as in SR).

- In $M^4{}_2$ in emitting the scattered quantum by the virtual electron (version A), the scattered γ'-quantum velocity is equal $c' = c_0\sqrt{1 + \hbar^2\omega^2/(\mathcal{E}_0{}^2 + 2\hbar\omega\mathcal{E}_0)}$ and exceeds c_0.

- In $M^4{}_2$ in the case of spontaneous transmutation of the virtual electron into the free electron with the following emission of the scattered γ'-quantum (version B) the scattered γ'-quantum velocity depends on the angle of its scattering and is equal $c' \sim c_0$ if the scattering occurs forward in the range of angles $\theta \ll \mathcal{E}_0/\hbar\omega$, and $c' \sim c_0(\hbar\omega/\mathcal{E}_0)$ for the back-scattering.

- In $M^4{}_2$ the Compton electron velocity v' trends to ∞ with $\hbar\omega \to \infty$. Faster-than-light motion of the forward-scattered electron begins from the energy of incident quantum $\mathcal{E}_0/\sqrt{2} \sim 360$ keV.

- In both the Minkowski spaces the motion of forward-scattered electron ($\alpha = 0$) corresponds to the motion of scattered γ'-quantum in the backward direction ($\theta = \pi$) with the energy $\hbar\omega' = \hbar\omega/(1 + 2\hbar\omega/\mathcal{E}_0) \sim \mathcal{E}_0/2 \sim 250$ keV if $\hbar\omega \gg \mathcal{E}_0$ (as in SR).

8 Turning to equations of quantum theory

Let us make clear how the basic equations of quantum theory (Schrödinger, Klein-Gordon-Fock and Dirac equations) may be written in the $M^4{}_2$-space. For this purpose we shall use the standard approach and pass on to the operator form for energy and momentum in the line 6 of Table 1 accordingly to the rule:

$$\mathcal{E} \to i\hbar(c_0/c)\partial/\partial t, \quad \mathbf{p} \to -i\hbar\nabla. \tag{47}$$

Here the operator for energy takes the well-known form [2], if $c = c_0$. The operator for momentum is standard [2]. Then for the free motion of a quantum particle we have the following equations.

The Schrödinger equation. Taking into consideration that in non-relativistic approximation with $v \ll c_0$ the velocity of light $c \sim c_0$ and the expression for kinetic energy $T = p^2/2m$ is the same for $M^4{}_2$ and $M^4{}_1$, we

16

find that the Schrödinger equation [2] will be the same in both the spaces:

$$\left(i\hbar\partial_t + \frac{\hbar^2}{2m}\triangle\right)\psi(t,\mathbf{x}) = 0, \tag{48}$$

where m is the mass of a particle, $\psi(t,\mathbf{x})$ is the wave function. As a result the non-relativistic quantum theory (the quantum mechanics) and the classical mechanics are the same for $M^4{}_2$ and $M^4{}_1$. The difference will appear in the relativistic range of motion. In $M^4{}_2$ this range begins with the energy $\mathcal{E} \geq \sqrt{2}\mathcal{E}_{0e} \sim 723$ keV for electron and $\mathcal{E} \geq \sqrt{2}\mathcal{E}_{0p} \sim 1.33$ GeV for proton. (The velocities of these particles will be equal or above $c_0 = 3 \cdot 10^{10}$ cm/s).

The Klein-Gordon-Fock equation. With the help of the dispersion relation (17) we obtain

$$\left(\frac{1}{c^2}\partial_{tt} - \triangle + \frac{m^2c_0{}^2}{\hbar^2}\right)\Phi(ct,\mathbf{x}) = 0. \tag{49}$$

The Dirac equation.

$$\left(i\gamma^0\frac{1}{c}\partial_t + i(\gamma^1\partial_x + \gamma^2\partial_y + \gamma^3\partial_z) - \frac{mc_0}{\hbar}\right)\Psi(ct,\mathbf{x}) = 0. \tag{50}$$

Here γ^0, γ^1, γ^2, γ^3 are the Dirac matrices, $\Phi(ct,\mathbf{x})$ and $\Psi(ct,\mathbf{x})$ are the wave functions. As distinct from $M^4{}_1$, in the $M^4{}_2$-space with $c \to \infty$ the equations (49), (50) are characterized by the appearance of solutions not depending on the time because the components with the derivative with respect to time vanish. If $c = c_0$, the equations (49), (50) go into SR equations [1, 28].

9   Discussion and conclusion

It has been considered the version of a mathematical theory that is similar to SR but differs from it in view of its being based on the metric of more general form (1). Here the velocity of light run through the continuous spectrum of values from $c_0 = 3 \cdot 10^{10}$ cm/s to ∞. It is believed from formally mathematical standpoint that the space with such a metric is 5-dimensional. It contains two Minkowski space: the first space $M^4{}_1$ on the hyper-plane c_0 with the local time $x^0 = c_0 t$, where SR is realized, and the second space $M^4{}_2$ with the non-local time $x^0 = \int_0^t c d\tau$, where realized is the theoretical version from the present work and the publications [3] - [5]. Some like ideas are contained in the well-known monograph of Pauli [7]. On the page 29 in discussing the Michelson experiment, Pauli notes that according to Abraham the velocity of light in frame K' moving together with the interferometer is equal

$$c' = c\sqrt{1 - \beta^2}. \tag{51}$$

17

This differs from the velocity of light c in laboratory frame K^2. According to Abraham the time dilatation is absent. The Abraham's viewpoint conforms to the result of Michelson experiment but contradicts the relativity principle because it leaves room for absolute motion [7].

It is interesting to note that if we find c from Abraham's formula and postulate $c' = c_0' = 3 \cdot 10^{10}$ sm/s, we just obtain the relations (2) and (3) of the present work that is in agreement with the principle of relativity. Thus, the Abraham's point of view turned out to be associated in an indirect way with the Finsler space (1) and with the presence of the two Minkowski spaces $M^4{}_1$ and $M^4{}_2$ in it. This is the simplest example of turning to spaces of such a type. However this simplicity makes deep sense as it is conditioned by fundamental properties of 3-space and time such as isotropy and homogeneity. The more complicated examples of non-homogeneous space-time with the metric $ds^2 = c^{-2N} \left\{ [cdt + (1 - N)tdc]^2 - \sum_j (dx^j - Nx^j dc/c)^2] \right\}$, where N is the number, $|c| < \infty$, $j = 1, 2, 3$, are considered in [27, 31, 32]. This metric enables one to introduce three Minkowski spaces: on the hyper-plane $c = c_0$=const with the time $x^0 = c_0{}^{1-N}t$, on the vectors $(x^0 = c^{1-N}t, x^j = (c^{-N}x, c^{-N}y, c^{-N}z))$ with the time $x^0 = c^{1-N}t$, and on the hyper-plane $t = t_0$=const with $x^0 = c^{1-N}t_0$. In the last case the role of time as a scalar parameter will play the velocity of light c. Motions in this space will happen beyond the conventional conception of time. At present it is not clear what the possibility of existing additional Minkowski spaces means, as well as whether this possibility has to do with the physical reality. It is a subject for further investigations.

In sum, we have shown that in the $M^4{}_2$-space it is possible to construct the theory, which admits faster-than-light motions of electromagnetic fields and particles with real masses. As a subgroup of symmetry, it contains the Poincaré group. Unlike motions described by SR in $M^4{}_1$, in the $M^4{}_2$-space it is possible to introduce the time similar to the universal Newton time on the trajectory of a particle. The particle mass does not depend on the velocity of its motion and is the fundamental constant as in classical mechanics. According to the Compton-effect kinematics in the $M^4{}_2$-space the scattered electron will move faster than $c_0 = 3 \cdot 10^{10}$ cm/s, if the incident γ-quantum energy exceeds 360 keV. For example, in the case of the annihilation quantum with the energy 511 keV (Na^{22}) and the propagation velocity $c = c_0$ the forward-scattered electron will be moving with the velocity $0.8c_0$ in the $M^4{}_1$-space and $1.3c_0$ in the $M^4{}_2$-space. This distinction (if it exists really) may be experimentally detected by means of measuring the flight-time of the

---

[2] Abraham, 1908, $\beta = V/c$ [7].

Compton electrons and annihilation γ-quantum on the base 100 cm long.

References

[1] Akhieser A.I. and Berestetsky V.B. Quantum Electrodynamics. Moscow: Nauka. - 1969. - 624.

[2] Shirokov Yu.M. and Yudin N.P. Nuclear Physics. Moscow: Nauka. - 1972. - 672.

[3] Kotel'nikov G.A. On the faster-than-light motions in electrodynamics. // Proceedings of XII International Conference on Selected Problems of Modern Physics. - Dubna: 2003. - D 1, 2- 2003-219. - P. 143-147; **physics/0311041**.

[4] Kotel'nikov G.A. On the possibility of faster-than-light motions in nonlinear electrodynamics. // Proceedings of Institute of Mathematics of NAS of Ukraine. - 2004. - V. 50. - Part 2. - P. 835-842; **physics/0405085**.

[5] Kotel'nikov G.A. On the electrodynamics with faster-than-light motion. // In Has the Last Word Been Said on Classical Electrodynamics? New Horizons, Editors A. Chubykalo, V. Onoochin, A. Espinoza, R. Smirnov-Rueda. Princeton: Rinton Press Inc. - 2004. - P. 71-81; **physics/0406010**.

[6] Landau L.D. and Lifshitz E.M. Mechanics. Moscow: Fizmatgiz. - 1958. - 207.

[7] Pauli W. Theory of Relativity. Moscow-Leningrad: Gostexizdat. - 1947. - 300.

[8] Landau L.D. and Lifshitz E.M. Theory of Field. Moscow: Nauka. - 1973. - 504.

[9] Logunov A.A. Lectures on Fundamentals of Relativity Theory. M.V. Lomonosov Moscow State University. - 1982. - 114.

[10] Buseman Herbert. The geometry of Finsler spaces. // Bulletin of the American Mathematical Society. - 1950. - V. 56. - January to December. - P. 5-16.

[11] Ikeda S. Some constructive comments on the theory of fields in Finsler spaces. // Lett. Nuovo Cimento. - 1978. - V. 21. - No 16. - P. 567-571.

[12] Efimov N.V. and Rosendorn E.R. Linear Algebra and Multidimensional Geometry. Moscow: Nauka. - 1970. - 528.

[13] Blokhintsev D.I. Note on the relativistic-invariant generalization of the concept of field. // JETP. - 1946. - V. 18. - Issue 6. - P. 480-482.

[14] Kirzhnits D.A. To the question on meson-nucleon interactions. // JETP. - 1954. - V. 27. - Issue 1(7). - P. 6-18.

[15] Terletsky Ya.P. Principle of causality and the second law of thermodynamics. // Dokl. Akad. Nauk (USSR). - 1960. - V. - 133. - No 2. - P. 329-332.

[16] Feinberg G. On the possibility of faster than light particles. // In Einstein Sbornik 1973. - Moscow: Nauka. - 1974. - P. 134-177; Phys. Rev. - 1967, - V. 159. - No 5. - P. 1089-1105.

[17] Bilaniuk O. and Sudarshan E. Particles beyond the light barrier. // In [16]. - P. 112-133; Physics Today. - 1969. - V. 22. - May. - P. 43-51.

[18] Recami E. Relativity theory and its generalization. // In Astrophysics, Quanta and Relativity Theory. Moscow: Izdatelstvo Mir. - 1982. - P. 53-128.

[19] Kirzhnits D.A. and Sazonov V.N. Superluminal motions in the special relativity theory. // In [16]. - P. 64-111.

[20] Sazonov V.N. Possible mass anisotropy and interaction of ultra-high-energy particles with electromagnetic field. // Nucl. Phys. (Russia). - 1972. - V. 15. - Issue 5. - P. 1060-1068.

[21] Chubykalo A., Espinoza A., Flores-Alvarado R., Gutierrez Rodriguez A. Helmholtz theorem and the v-gauge in the problem of superluminal and instantaneous signals in classical electrodynamics. Foundation Phys. Lett. // - 2006. - V. 19. - No 1. - P. 37-49.

[22] Chubykalo A.E. and Vlaev S.J. Necessity of simultaneous co-existence of instantaneous and retarded interactions in classic electrodynamics. // Int. J. Mod. Phys. A. - 1999. - V. 14. - Issue 24. - P. 3789-3798.

[23] Chubykalo A.E. and Smirnov-Rueda R. Action at a distance as a full-value solution of Maxwell equations: basis and application of separated potential's method. // Phys. Rev. E. - 1996. - V. 53. - Issue 5. - P. 5373-5381.

[24] Barut A.O. and Haugen R.B. Theory of the conformally invariant mass. // Annals of Phys. - 1972. - V. 71. - P. 519-541.

[25] Poincáre H. Sur la dynamique de l'electron. // In Principle of Relativity. - Moscow: Atomizdat. - 1973. - P. 118-167.

[26] Kotel'nikov G.A. Group properties of wave equation with the non-invariant velocity of light. // Theor. Math. Phys. (Russia). - 1980. - V. 42. - No 1. - P. 139-144.

[27] Kotel'nikov G.A. Equations of electrodynamics with non-invariant velocity of light. // Izv. Vuz. Fiz. - 1981. - No 10. - P. 46-51; Russian Phys. J. - 1981. - V. 24. - No 10. - P. 938-942.

[28] Fushchich W.I. and Nikitin A.G. Symmetry of Maxwell Equations. - Kiev: Naukova Dumka. - 1983. - 200.

[29] Levich V.G. Course of Theoretical Physics. - V. 1. - Moscow: Nauka. - 1969. - 911.

[30] Bogoliubov N.N. and Shirkov D.V. Introduction into the Theory of Quantized Fields. - Moscow: Nauka. - 1973. - 416.

[31] Kotel'nikov G.A. Quasars as objects with apparent superluminal expansion. // Survey of the Kurchatov Atomic Energy Institute. - Moscow. - 1979. - 36 P.

[32] Kotel'nikov G.A. Realization of Dirac hypothesis for the dependence of fundamental constants on time in violating the invariance of the speed of light. // Izv. Vuz. Fiz. - 1979. - No 9. - P. 93-95; Russian Phys. J. - 1979. - V. 22. - No 9. - P. 999-1001.